Preferred Networks

BNN verification dataset for Max-SAT Evaluation 2020

Masahiro Sakai (sakai@preferred.jp)

NII Shonan Meeting (No.180) on "The Art of SAT"
Oct. 2023

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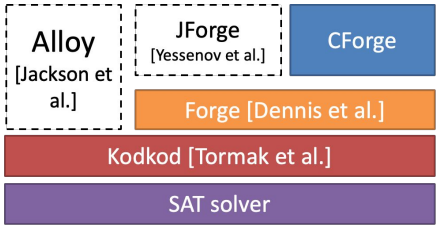
Some of my SAT/SMT-related activities

1. Japanese Translation of "Software Abstractions"
A book about Alloy (a SAT-based finite model finder) and Formal Methods

2. CForge: Model-Checking C Programs against JML-like Specification Language



(Work in Toshiba)

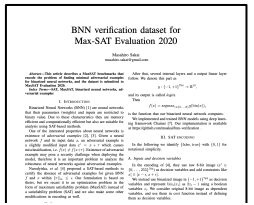
Also we combined BMC and UNSAT-core enumeration for precondition inference

3. toysat
SAT/Max-SAT/PB/SMT/QBF/MILP solver written in Haskell

A hobby project for learning solver algorithms, but submitted to some past PB / Max-SAT / SMT competitions.

github:msakai/toysolver

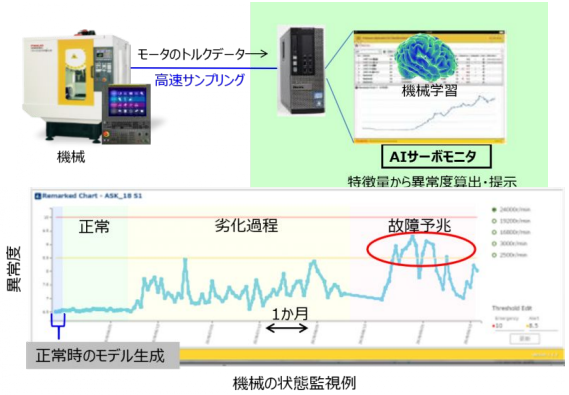
4. Binarized Neural Network verification instances for Max-SAT Evaluation 2020

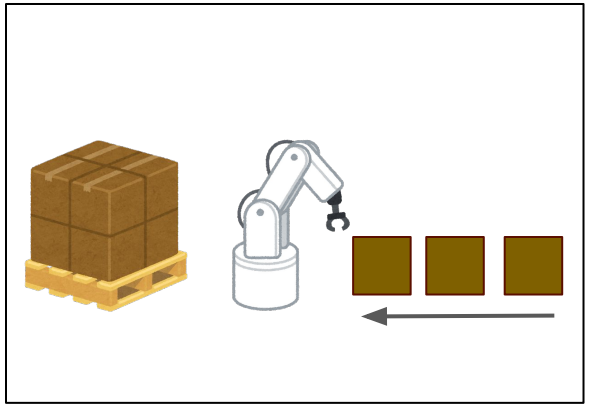


github:msakai/bnn-verification

Personal Motivation

I have been using SAT/SMT and also implementing my own solver.
Then, I learned machine learning, joined Preferred Networks in 2017, and worked on projects that uses (mainly) machine learning.





So, I'm interested in the overlapping area of machine learning and SAT/SMT.

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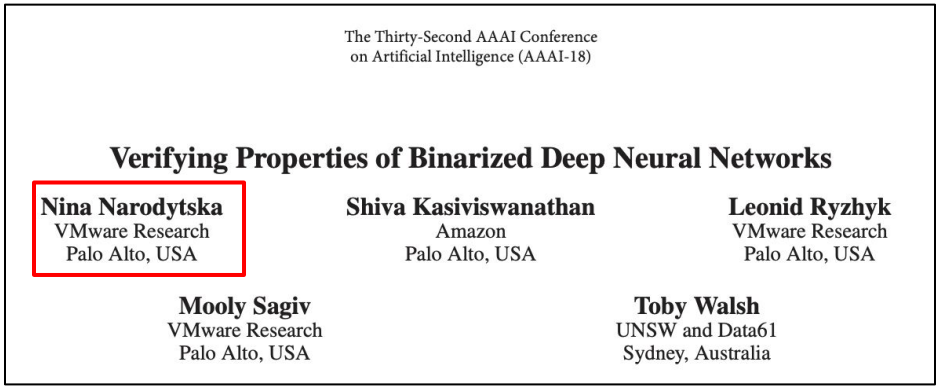
Acknowledgement

Thank you to the authors of [Narodytska+, AAAI-18] and organizers of the MaxSAT Evaluation 2022.

This work is based on the paper:



The Thirty-Second AAAI Conference
on Artificial Intelligence (AAAI-18)

Verifying Properties of Binarized Deep Neural Networks

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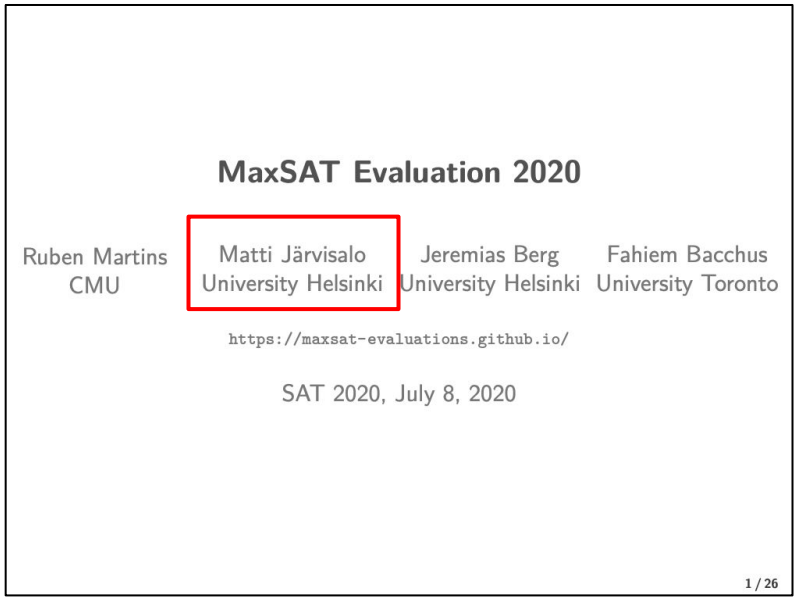
https://www.aaai.org/ocs/index.php/AAAI/AAAI18/paper/view/16898



MaxSAT Evaluation 2020

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https://maxsat-evaluations.github.io/

SAT 2020, July 8, 2020

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https://maxsat-evaluations.github.io/2020/mse20-talk.pdf

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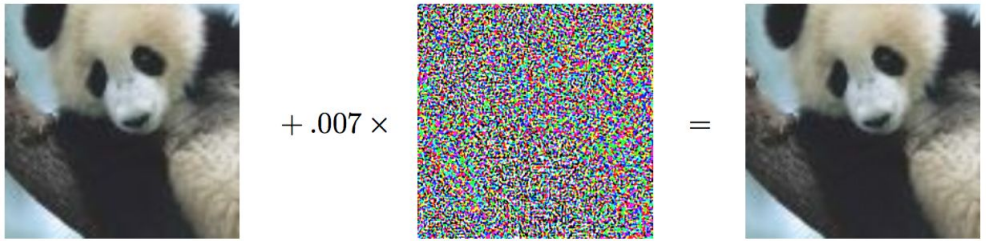
Outline

- 1. Background: DNN verification
- 2. Background: Binarized Neural Networks (BNN)
- 3. (Max-)SAT-based verification
  - Models
  - SAT-encoding
  - Cost Function and Soft Clauses
- 4. Result in the Max-SAT Evaluation 2020
- 5. Conclusion

1. Background on DNN verification

Adversarial perturbations

- Deep neural networks (DNN) achieved impressive performance in various tasks, But …



- "Explaining and Harnessing Adversarial Examples" [Goodfellow+, ICLR 2015]
- "Intriguing properties of neural networks" [Szegedy+, ICLR 2014]
- …

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Formal guarantees on DNN behavior

- Such non-robustness may result in **unsafe systems**, or **restrict the use of DNNs in safety-critical applications**.
  - e.g. autonomous vehicles, airborne collision avoidance systems, deep-brain stimulation against epileptic seizures, robots, etc.
- Hence, there is a need for **formal guarantees about their behavior**.
- There are lots of research using e.g. SAT, SMT, CEGAR, MILP

Basic approach

For example, a property called **local adversarial robustness** can be checked by SAT/SMT by checking if

$$\|\tau\| \leq \varepsilon \land f(x+\tau) \neq f(x)$$

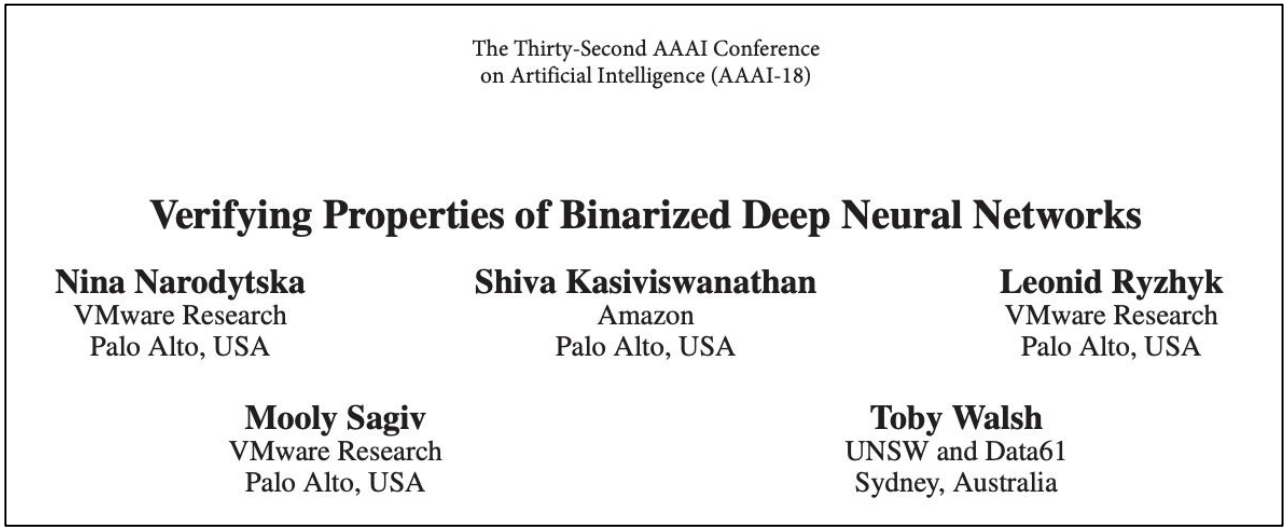
is satisfiable.

However, neural network **f** is often **very large**, **complex**, and **contains massive amounts of floating point operations.**
⇒ Difficulty for SAT/SMT solvers

There are many researches for overcoming the problem.

Focusing on specific type of NN

- [Narodytska+, AAAI-18] is one such research
- They focus on a specific type of neural networks called Binalized Deep Neural Networks (BNN)



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Verifying Properties of Binarized Deep Neural Networks

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https://www.aaai.org/ocs/index.php/AAAI/AAAI18/paper/view/16898

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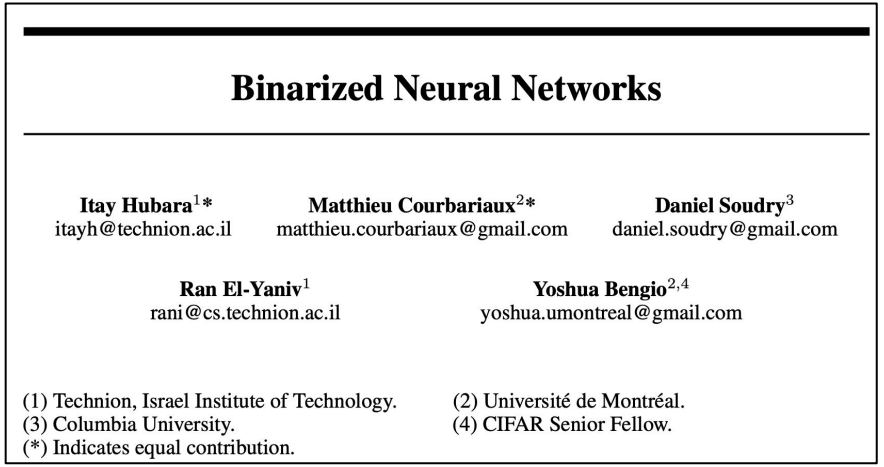
2. Background: Binarized Neural Networks (BNN)

Binarized Neural Networks (BNN)

- **Large number of floating point operations** are computationally expensive and require big memory
- Usually **float32** is used
- Quantization is a technique to use **reduces bit-width floating point**, or **small bit-width integer** instead
- In the extreme case, binary values {-1, +1} is used

- One such attempt is Binarized Neural Networks [Hubara+, NIPS 2016]



Binarized Neural Networks

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https://papers.nips.cc/paper/6573-binarized-neural-networks

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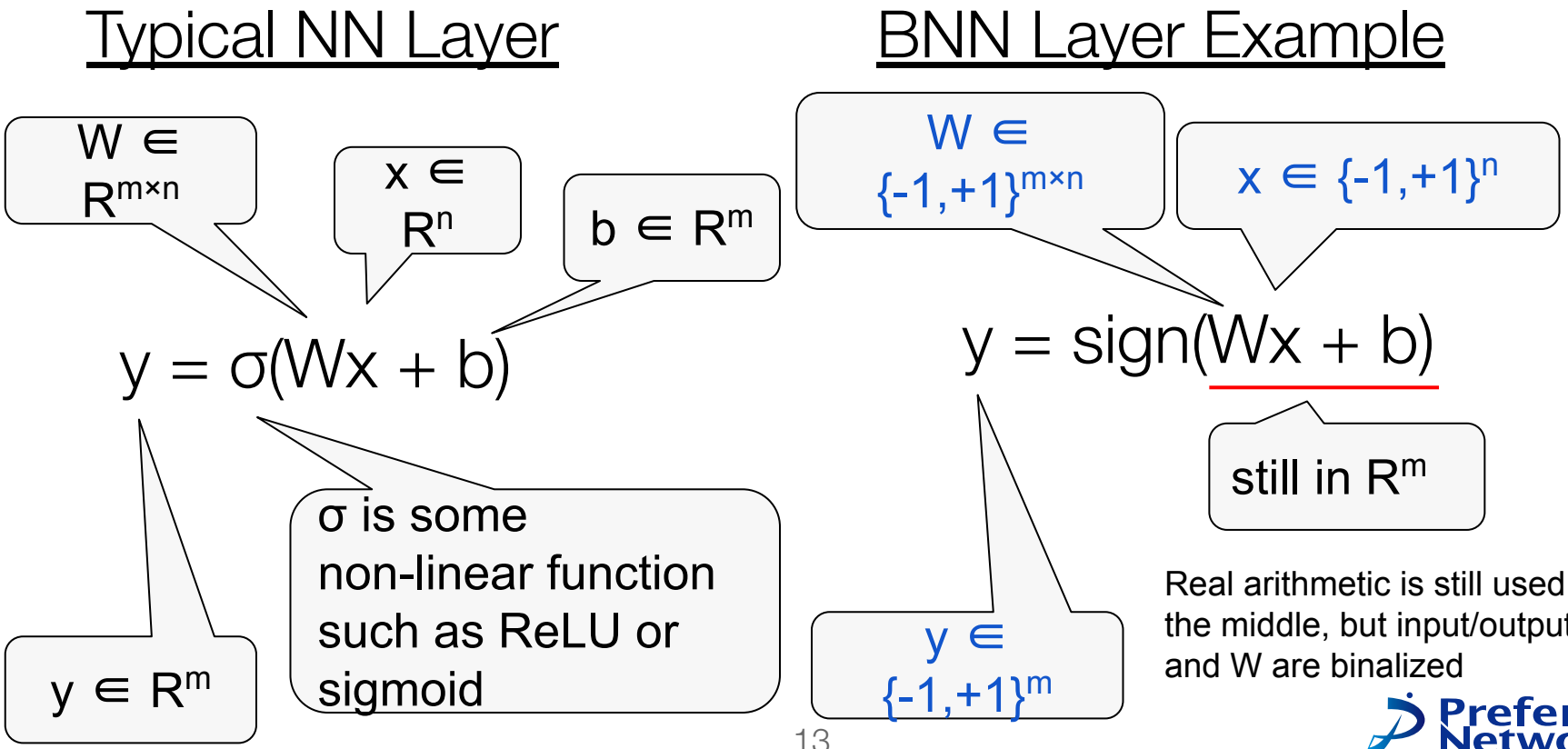
Binarized Neural Networks (BNN)



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Binarized Neural Networks (BNN)

- Binarization is for performance (e.g. computation time, memory usage, energy consumption, circuit area, …)

- But, at the same time, it is very convenient for SAT solvers

3. (Max-)SAT-based verification

Our approach is based on [Narodytska+, AAAI-18]
Since many parts are the same, I will talk only the overall idea and the details of the difference.

Decision problem ⇒ Optimization probem

[Narodytska+, AAAI-18] verifies absence of adversarial examples (among other properties) by showing

$$\|\tau\|_\infty \leq \varepsilon \ \wedge \ f(x_{orig}) \neq f(x_{orig} + \tau)$$

is UNSAT for given f, x, and ε.

However, this can be generalized to an optimization problem

$$\text{min. } \|\tau\|_\infty \quad \text{s.t. } \ f(x_{orig}) \neq f(x_{orig} + \tau)$$

i.e. finding minimum adversarial perturbation.

(Actually, we encode the objective function as a set of weighted soft clauses to use Max-SAT solvers)

Models

High-level structure of our model and input data

- High-level structure of our model:

$$f(x) = \mathrm{argmax}_{c \in \{0,\ldots,9\}} g(\mathrm{bin}(x))_c$$

- Input image $x \in \{0, \ldots, 255\}^{784}$
  - 784 (= 28×28) dimension vector of 8-bit integers
- Binarized image $\mathrm{bin}(x) \in \{-1, +1\}^{784}$
  - x is converted to binarized image $\{-1, +1\}^{784}$
  - Note that the binarization threshold is different depending on components (i.e. pixel locations)

Internal and output layers

- High-level structure of our model:

$$f(x) = \mathrm{argmax}_{c \in \{0,\ldots,9\}} g(\mathrm{bin}(x))_c$$

- We denote composition of internal layers and a output linear layer as $g : \{-1, +1\}^{784} \to \mathbb{R}^{10}$
- The output $g(\mathrm{bin}(x)) \in \mathbb{R}^{10}$ is called **logits**

Internal and output layers

- High-level structure of our model:

$$f(x) = \underline{\mathrm{argmax}_{c \in \{0,\ldots,9\}}} g(\mathrm{bin}(x))_c$$

- $g(\mathrm{bin}(x)) \in \mathbb{R}^{10}$ is called **logits**
- **argmax** compare the logits and choose a class c with maximal logit

Implementation and Training

- High-level structure of our model:

$$f(x) = \mathrm{argmax}_{c \in \{0,\ldots,9\}} g(\mathrm{bin}(x))_c$$

- We implemented the model and trained using deep learning framework Chainer

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Implementation and Training

- **Trained three models for three datasets**
  - MNIST and its two variants: MNIST-rot and MNIST-back-image

100 sample images from these dataset

MNIST

MNIST-rot

MNIST-back-image

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SAT encoding

Inputs and decision variables

$$f(x) = \mathrm{argmax}_{c \in \{0,\ldots,9\}} g(\mathrm{bin}(x))_c$$

- [Narodytska+, AAAI-18] uses perturbation τ as decision variables and add constraints like $\tau \in [-\varepsilon, +\varepsilon]^{784}$ and $x_{orig}+\tau \in [0, 255]^{784}$ which involve integer addition and comparison

- We instead use binarized image $z = \mathrm{bin}(x_{orig}+\tau) \in \{-1, +1\}^{784}$ as decision variables
  - Once you find a desired binarized image, it is easy to construct smallest perturbation in original image space
  - Reduced number of decision variables and constraints

Relationship between inputs and logits

$$f(x) = \mathrm{argmax}_{c \in \{0,\ldots,9\}} g(\mathrm{bin}(x))_c$$

- The g part is encoded to CNF in a way similar to [Narodytska+, AAAI-18]
  - Detail omitted here, but can be encoded to clauses and cardinality constraints
- We use **totalizer** [Bailleux, CP 2003] instead of **sequential counter** [Sinz+, CP 2005] for encoding cardinality constraints to reduce formula size

Relationship between logits and outputs

$$f(x) = \underline{\mathrm{argmax}_{c \in \{0,\ldots,9\}}} g(\mathrm{bin}(x))_c$$

- Let logits := g(bin(x))
- Let y := $\{y_c\}_c$ be ten boolean variables such that $y_c \Leftrightarrow f(x)=c$

- Since y is one-hot vector
  - $\Sigma_c\, y_c = 1$
- If y_c is true, c-th logit must be largest
  - $y_c \rightarrow \underbrace{\mathrm{logits}_c \geq \mathrm{logits}_{c'}}$ for all c′.

A cardinality constraints if we expand the definition of logits

(Conditional) cardinality constraints are encoded using totalizer

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Relationship between logits and outputs

$$f(x) = \underline{\mathrm{argmax}_{c \in \{0,\ldots,9\}}} g(\mathrm{bin}(x))_c$$

- Let logits := g(bin(x))
- Let y := $\{y_c\}_c$ be ten boolean variables such that $y_c \Leftrightarrow f(x)=c$

- Finally, we want the input to be misclassified
- If the (true) label of the image is c, we add $(\neg\, y_c)$

Cost Function and Soft Clauses

Turning cost functions into soft constraints

- To find the smallest adversarial perturbation, we have to penalize modification from x_is
- Our decision variables are binarized pixels z_is instead of τ_is
- Therefore, the basic idea is to add the following as soft constraints with appropriate weights

$$z_i = \mathrm{bin}_i(x_{\mathrm{orig},i})$$

This is a constant

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Some preparation

- Let τ_i be the change of x_i
  - $x_i = x_{orig,i} + \tau_i$
- Let δ_i be the smallest perturbation to change z_i
  - $bin_i(x_{orig,i} + \delta_i) \neq bin_i(x_{orig,i})$
- Since we are minimizing τ, we can let
  - τ_i = (if $z_i = bin_i(x_{orig,i})$ then 0 else δ_i)

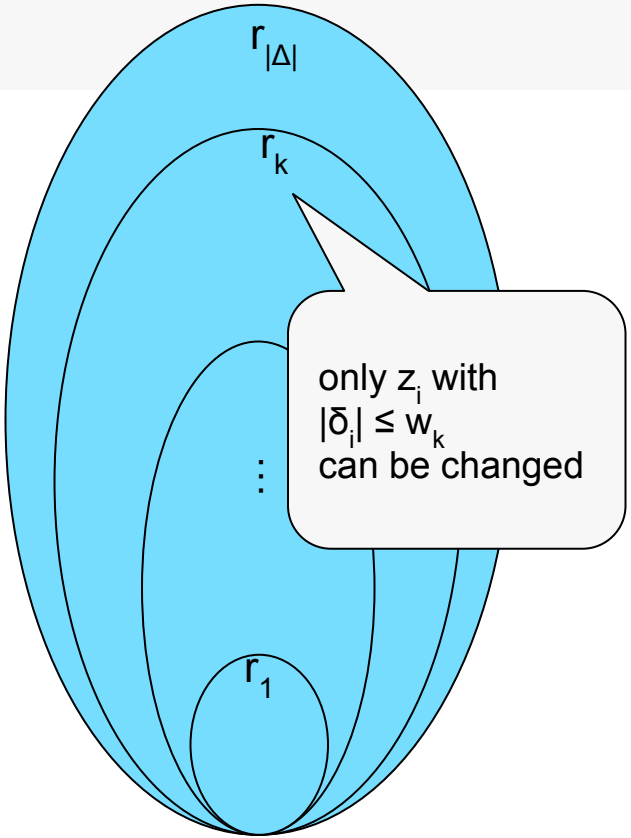
L_p-norm case for $p \neq \infty$

- $\|\tau\|_p = (|\tau_1|^p + \ldots + |\tau_n|^p)^{(1/p)}$
- Since $(-)^{(1/p)}$ is monotone, minimizing $\|\tau\|_p$ is equivalent to minimizing $|\tau_1|^p + \ldots + |\tau_n|^p$
- Since τ_i = (if $z_i = \mathrm{bin}_i(x_{\mathrm{orig},i})$ then 0 else δ_i), it is equivalent to have $(z_i = \mathrm{bin}_i(x_{\mathrm{orig},i}))$ as a soft constraint with weight $|\delta_i|^p$

L_∞-norm case

- $\|\tau\|_\infty = \max(|\tau_1|, \ldots, |\tau_n|)$

- Let $\Delta = \{ |\delta_i| \} = \{w_1 < \ldots < w_{|\Delta|}\}$
- Introduce relaxation variables $\{r_k\}_{k \in 1..|\Delta|}$ with
  - $\neg r_k \longrightarrow (z_i = \mathrm{bin}_i(x_{\mathrm{orig},i}))$
    for all i with $|\delta_i| = w_k$
  - $r_k \longrightarrow r_{k-1}$ *(they are lower closed)*
  - $(\neg r_k)$ with cost $(w_k - w_{k-1})$



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Result in the Max-SAT Evaluation 2020

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Instance generation and submission

- Average size of generated instance
  - #variables = 1.8 M
  - #clauses = 132 M
  - (uncompressed) WCNF filesize = 3.6 GB
- Submitted 60 instances
  - 3 dataset and trained model
  - 2 images for each of 10 digit class
  - only used L_∞-norm setting

5 among 60 submitted instances were used

Instance	Image	Label
bnn_mnist_7_label9_adversarial_norm_inf_totalizer.wcnf.gz		9
bnn_mnist_back_image_32_label3_adversarial_norm_inf_totalizer.wcnf.gz		3
bnn_mnist_rot_16_label5_adversarial_norm_inf_totalizer.wcnf.gz		5
bnn_mnist_rot_8_label1_adversarial_norm_inf_totalizer.wcnf.gz		1
bnn_mnist_back_image_73_label5_adversarial_norm_inf_totalizer.wcnf.gz		5

Result: Complete track (Weighted)

https://maxsat-evaluations.github.io/2020/rankings.html

Instance	maxino-pref	maxino	Pacose	UWrMaxSat	MaxHS	QMaxSAT	RC2-B / RC2-A / smax-minisat / smax-mergesat
…	270.62	269.06	402.17	648.45	991.52	141.42	3600.0
…	279.84	277.76	1101.24	795.81	1733.77	1729.06	
…	367.28	367.06	221.87	657.69	1006.6	704.83	
…	84.87	84.06	347.71	588.25	1083.57	3600.0	
…	2215.51	2232.61	3600.0	3600.0	3600.0	3600.0	

Result: optimal objective values

Very small $\|\tau\|_\infty$ is enough ⇒ Those models are not robust at all… 😢

Instance	Image	Label	Minimal $\|\tau\|_\infty$
bnn_mnist_7_label9_adversarial_norm_inf_totalizer.wcnf.gz		9	1
bnn_mnist_back_image_32_label3_adversarial_norm_inf_totalizer.wcnf.gz		3	2
bnn_mnist_rot_16_label5_adversarial_norm_inf_totalizer.wcnf.gz		5	1
bnn_mnist_rot_8_label1_adversarial_norm_inf_totalizer.wcnf.gz		1	1
bnn_mnist_back_image_73_label5_adversarial_norm_inf_totalizer.wcnf.gz		5	4

Some slides from organizer's talk

MaxSAT Applications

- Many real-world applications can be encoded to MaxSAT:
  - Software package upgradeability
  - Error localization in C code
  - Haplotyping with pedigrees
  - Verification of binarized networks
  - ...
- MaxSAT algorithms are **very effective** for solving real-word problems

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New benchmarks

- Verification of binarized neural networks
- Maximum probability minimal cut sets
- Data flow security weaknesses of reconfigurable scan networks
- Coalition Structure Generation
- Role-Based Access Control maintenance
- Synthesis of minimal hardware exploits
- Most compatible phylogenetic trees over multi-state characters
- Railway timetabling
- Single-machine scheduling
- Analysis of large networks
- Set covering problems from Italian railways
- Set covering problems from Steiner Triple Systems
- Program disambiguation
- Inference of tumor evolutionary history
- User authorization query problems

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https://maxsat-evaluations.github.io/2020/mse20-talk.pdf

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Some slides from organizer's talk

New benchmarks

- MaxSAT is being used in many applications!
  - 15 different domains
- We received a lot of new benchmarks:
  - **5,091 new benchmarks!**
- Benchmark size is increasing significantly:
  - **81 GB** (after gzip compression)
- MaxSAT solvers can solve very large problems:
  - More than 20 **million variables** and 130 **million clauses**

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Complete track: Weighted

- Improvements with respect to best solver at MSE 2019:
  - RC2: 417 solved instances, 205.88 seconds (average)
- VBS solves 479 instances:
  - Smaller gap than in 2019
- Large benchmarks can be solved:
  - pa-3.wcnf.gz:
    - 19,264,629 variables, 22,951,677 clauses
    - Solved by MaxHS in 731.64 seconds!
  - bnn_mnist_back_image_73_label5_adversarial_norm_inf_totalizer.wcnf.gz
    - 1,825,399 variables, 132,682,679 clauses
    - Solved by Maxino in 2215.51 seconds!

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https://maxsat-evaluations.github.io/2020/mse20-talk.pdf

Conclusion

Some concluding remarks (1)

- **Max-SAT solvers can solve big instances (as mentioned by organizer)**
  - Even w/ 1.8M variables and 132M constraints
  - I thought that the problem size MaxSAT solver can handle was limited (compared to SAT), but it can solve larger than I thought!
- **Disclaimer:**
  - This is based on the 2018 paper and the work was done on 2020, so some part of the talk may already be out of date

Some concluding remarks (2)

- **Submitting problems was an interesting and enjoyable experience for me**

- **Future interaction of Machine Learning and SAT/SMT**
  - This is still a toy problem and model (MNIST is like “Hello World” in machine learning)
  - But I hope these two areas to have more fruitful interactions in the future

Any Questions or Comments?

Source code and dataset are available at:
github.com/msakai/bnn-verification

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Making the real world computable